Progression in Written Calculations:

**ADDITION**

**Key Vocabulary:**

<table>
<thead>
<tr>
<th>add</th>
<th>addition</th>
<th>and</th>
<th>plus</th>
<th>count on</th>
<th>more</th>
</tr>
</thead>
<tbody>
<tr>
<td>increase</td>
<td>makes</td>
<td>sum</td>
<td>total</td>
<td>altogether</td>
<td></td>
</tr>
</tbody>
</table>

**EARLY STAGE:**

At this stage, children are not expected to use paper and pencil procedures for addition. Their experience of these operations will be a mixture of practical, oral and mental work.

| Recognise numbers 0 to 10
|---|
| Count reliably up to 10 everyday objects
| Count one more than a number
| Begin to add two small groups of objects together
| Count in ones to add numbers together

Before pupils can move to recording 3+2, they will need experience of practical addition, and an ability to respond to mathematical vocabulary practically. Children use signs and symbols to record practical work.
**Key Stage 1**

At this stage, pupils need experience of adding using concrete objects and pictorial representations in a wide variety of contexts. Pupils will use number lines extensively to develop mental calculations which will assist them when recording. Children begin to use blank number lines to support their own calculations.

<table>
<thead>
<tr>
<th>Begin to read and use the + and = signs to record practical work and mental calculations in a number sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Concrete objects and pictorial representations" /></td>
</tr>
<tr>
<td>3 + 2 = 5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Use concrete objects and pictorial representations to assist in adding one and two digit numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image2" alt="Children using number lines" /></td>
</tr>
<tr>
<td>1 + 2 = 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Children begin to use prepared number lines to support their own calculations to add one and two digit numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Number line counting up in 1's" /></td>
</tr>
<tr>
<td>8 + 5 = 13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Know by heart all number bonds with a total of 10 or 20.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4" alt="Number bonds" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Counting on in twos, fives and tens.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5" alt="Counting in twos" /></td>
</tr>
</tbody>
</table>
**Partition** 2-digit numbers in order to add at a later stage

Recall all number bonds with a total of 10 and 20 fluently and use this to use related number bonds to 100.

Continue to add and subtract numbers using concrete objects and pictorial representations to add:
- a 2-digit number and ones
- a 2-digit number and tens
- two 2-digit numbers
- adding three 1-digit numbers

Know that addition can be done in any order.

To put the biggest number first when counting on.
Add two single-digit numbers, counting up in ones

\[
\begin{align*}
9 + 8 &= 17 \\
&= \underbrace{+1}_{9} + \underbrace{+1}_{10} + \underbrace{+1}_{11} + \underbrace{+1}_{12} + \underbrace{+1}_{13} + \underbrace{+1}_{14} + \underbrace{+1}_{15} + \underbrace{+1}_{16} + \underbrace{+1}_{17}
\end{align*}
\]

Continue Counting up in 1’s

Add two single-digit numbers that bridge 10

\[
\begin{align*}
8 + 7 &= 15 \\
&= \underbrace{+2}_{8} + \underbrace{+1}_{10} + \underbrace{+1}_{11} + \underbrace{+1}_{12} + \underbrace{+1}_{13} + \underbrace{+1}_{14} + \underbrace{+1}_{15} \\
\end{align*}
\]

Using knowledge of number bonds to 10 (8+2=10)

Counting on in tens and ones

\[
\begin{align*}
14 + 13 &= 27 \\
&= \underbrace{+10}_{14} + \underbrace{+1}_{24} + \underbrace{+1}_{25} + \underbrace{+1}_{26} + \underbrace{+1}_{27}
\end{align*}
\]

Adding the tens and ones in one jump

\[
\begin{align*}
24 + 13 &= 37 \\
&= \underbrace{+10}_{24} + \underbrace{+3}_{34} + \underbrace{+3}_{37}
\end{align*}
\]

Adding the tens in one jump and the ones in one jump

\[
\begin{align*}
24 + 43 &= 67 \\
&= \underbrace{+20}_{43} + \underbrace{+4}_{63} + \underbrace{+4}_{67}
\end{align*}
\]

Remember to start from the biggest number

Add two 2-digit numbers that bridge 10

\[
\begin{align*}
32 + 43 &= 75 \\
&= \underbrace{+20}_{43} + \underbrace{+7}_{63} + \underbrace{+5}_{70} + \underbrace{+5}_{75}
\end{align*}
\]

Using knowledge of number bonds to 10 (3+7=10)

Children will continue to use empty number lines with increasingly large numbers, and counting on from the largest number irrespective of the order of the calculation

\[
\begin{align*}
38 + 86 &= 124 \\
&= \underbrace{+30}_{86} + \underbrace{+4}_{116} + \underbrace{+4}_{120} + \underbrace{+4}_{124}
\end{align*}
\]

Start from the biggest number

Using knowledge of number bonds to 10 (6+4=10)
Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate:

$$49 + 73 = 122$$

Horizontal partitioning
Begin to record mental calculations using partitioning, splitting the tens and ones, and recombing:

$$13 + 46 = (10 + 40) + (3 + 6)$$
$$= 50 + 9$$
$$= 59$$

Column partitioning
Partitioning both numbers into tens and ones where ones are placed under ones and tens under tens to prepare children for formal columnar methods:

$$13 + 46 = 10 + 3$$
$$40 + 6$$
$$50 + 9 = 59$$

Round 49 to the nearest ten (50) then take-away the extra 1 that was added.
**Key Stage 2**
During Key Stage 2, children should become increasingly confident using the formal columnar method of addition. In Year Three they should be using 3 digit numbers, in Year 4 they should be using 4 digit numbers and in Year Five they should be able to add numbers with more than 4 digits. By Year Six this method should be completely embedded. Children should also be able to add decimals in the context of measure (money, length etc).

### Expanded column method
**Adding the ones first, then tens, then the hundreds**

<table>
<thead>
<tr>
<th></th>
<th>67</th>
<th>126</th>
</tr>
</thead>
<tbody>
<tr>
<td>+43</td>
<td>10 (7+3)</td>
<td>10 (6+4)</td>
</tr>
<tr>
<td></td>
<td>100 (60+40)</td>
<td>70 (20+50)</td>
</tr>
<tr>
<td></td>
<td>110</td>
<td>100 (100+0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>180</td>
</tr>
</tbody>
</table>

**Record mental calculation**

### Use expanded column method when adding money, beginning with decimals that require no carrying and then move onto carrying the tenths or hundredths only

<table>
<thead>
<tr>
<th></th>
<th>£ 2.50</th>
<th>£ 3.00 (£2.00 + £1.00)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+£ 1.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td>£ 1.20 (50p + 70p)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>£ 4.25</td>
<td></td>
</tr>
</tbody>
</table>

### Compact column method without carrying
Children add from least significant number without carrying

<table>
<thead>
<tr>
<th></th>
<th>H T U</th>
</tr>
</thead>
<tbody>
<tr>
<td>342</td>
<td>+ 57</td>
</tr>
<tr>
<td>399</td>
<td></td>
</tr>
</tbody>
</table>

### Add the ones first, then the tens and lastly, the hundreds

### Compact column method, carrying below the line (ONES only)
Children will begin with carrying only the ones below the line. 5+8 = 13. Record the 3 in the ones column, carrying the 1 ten.

<table>
<thead>
<tr>
<th></th>
<th>H T U</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>625</td>
</tr>
<tr>
<td>+48</td>
<td></td>
</tr>
<tr>
<td>673</td>
<td>1</td>
</tr>
</tbody>
</table>
### Compact column method, carrying below the line (TENS only)

<table>
<thead>
<tr>
<th>H</th>
<th>T</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>+</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>7 1</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

Children carry only the tens below the line. 2 tens + 9 tens = 11 tens. Record 1 ten in the tens column, carrying the 10 tens.

### Compact column method, carrying ones, tens and hundreds

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>587</td>
<td>3587</td>
<td></td>
</tr>
<tr>
<td>+ 475</td>
<td>+ 675</td>
<td></td>
</tr>
<tr>
<td>1062</td>
<td>4262</td>
<td></td>
</tr>
</tbody>
</table>

### Carrying decimals, begin with decimals to 1 decimal place and then moving to 2 decimal places

- **Carrying tenths only**
  - 32.5 km
  - + 54.6 km
  - **87.1 km**

### Carrying both tenths and hundredths

- £6.72
- £8.56
- + £2.33
- **£17.61**

### Compact column method, carrying ones, tens, hundreds and thousands

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7648</td>
<td>6584</td>
<td>42</td>
</tr>
<tr>
<td>+ 1486</td>
<td>+ 5848</td>
<td>6432</td>
</tr>
<tr>
<td>9134</td>
<td>12432</td>
<td>786</td>
</tr>
</tbody>
</table>

Adding more than two numbers

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
<td>111</td>
<td>3</td>
</tr>
<tr>
<td>+ 4681</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11944</td>
<td></td>
<td></td>
</tr>
<tr>
<td>121</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Add decimals with different numbers of places and different numbers of digits

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>124.9</td>
<td>124.9</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>86.3</td>
<td>117.25</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>211.2</td>
<td>242.15</td>
</tr>
<tr>
<td>111</td>
<td>11</td>
</tr>
</tbody>
</table>
**Progression in Written Calculations:**

**SUBTRACTION**

Key Vocabulary:

<table>
<thead>
<tr>
<th>take-away</th>
<th>count back</th>
<th>subtract</th>
<th>less</th>
</tr>
</thead>
<tbody>
<tr>
<td>fewer</td>
<td>minus</td>
<td>difference</td>
<td>difference between</td>
</tr>
</tbody>
</table>

**EARLY STAGE 1:**

At this stage, children are not expected to use paper and pencil procedures for subtraction. Their experience of these operations will be a mixture of practical, oral and mental work.

<table>
<thead>
<tr>
<th>Recognise numbers 0 to 10</th>
<th>0 1 2 3 4 5 6 7 8 9 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count back from 10</td>
<td>10 9 8 7 6 5 4 3 2 1 0</td>
</tr>
<tr>
<td>Count one less than a number</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>Begin to subtract one small group from another</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Using pictures or number tracks to subtract one group from another and counting what is left over.
**KEY STAGE 1**

Before pupils can move to recording 5-2, they will need experience of practical subtraction, and an ability to respond to mathematical vocabulary practically. Children use signs and symbols to record practical work only when they are ready. At this stage, pupils use number lines extensively to develop mental calculations which will assist them when recording. Children begin to use blank number lines to support their own calculations.

| Begin to use the - and = signs to record practical work and mental calculations in a number sentence | ![Image](image1) leaves 3  
5 - 2 = 3 |
|---|---|
| Children begin to understand that subtraction is not commutative | 6 - 2 = 4  
2 - 6 = 4 \(\times\) |
| Subtract 1 and 2 digit numbers to 20 – including 0. | 12 - 4 = 8  
15 - 5 = 10 |
| Relationship to addition: Use models and images to show the inverse relationship between addition and subtraction.  
Use this to solve missing number problems. | ![Image](image2)  
5 - 2 = 3  
3 + 2 = 5 |
| Recall subtraction facts to 20 | ![Image](image3)  
10 - 2 = 8  
20 - 2 = 18 |
Continue to add and subtract numbers using concrete objects and pictorial representations to add:
- a 2-digit number and ones
- a 2-digit number and tens
- two 2-digit numbers

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>3</th>
<th>-</th>
<th>2</th>
<th>=</th>
<th>1</th>
</tr>
</thead>
</table>

Children begin to use **prepared** number lines to support their own calculations, beginning with counting back in ones.

- **Counting back in 1’s**

Using a **prepared** number line to find the difference by counting up.

- The numberline is used to show that 7 - 4 means the ‘difference between 7 and 4’ or ‘the difference between 4 and 7’ and how many jumps they are apart.

Take – away numbers 0 - 20 from 20. Use these to use related facts up to 100.

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

**Because I know:**

- $10 - 3 = 7$
- $100 - 30 = 70$
Count forwards and backwards in 1’s, 2’s, 5’s and 10’s

Empty number lines to count back, first counting back in tens and ones

29 - 14 = 15

Empty number lines to count back, counting back in tens and the ones in one jump

29 - 14 = 15

Empty number lines to count back, counting back in tens and the ones in one jump

47 - 23 = 24

Subtract two 2-digit numbers that bridge 10

42 - 27 = 15

Using knowledge of number bonds to 10

(7+3=10) (10-3 = 7)

Empty number lines to count up

42 - 39 = 3

Calculations close together

It can be more efficient to count on if the numbers involved
in the calculation are close together or if the numbers involved in the calculation are near to multiples of 10, 100 etc.

\[
\begin{align*}
62 - 31 &= 31 \\
\end{align*}
\]

\[
\begin{align*}
+10 &+10 \quad +10 \\
31 &41 \quad 51 \quad 61 \quad 62 \\
\end{align*}
\]

Near to multiples of 10, 100 etc

\[
\begin{align*}
50 - 21 &= 29 \\
\end{align*}
\]

\[
\begin{align*}
+10 &+10 \quad +10 \quad -1 \\
21 &31 \quad 41 \quad 50 \quad 51 \\
\end{align*}
\]

Subtracting 9 or 11, 19 or 21 etc

Making decisions about counting up or back

With practice, children will need to decide whether to count back or forward and which method is more efficient to use.

Empty number lines with large numbers to count on
Children will continue to use empty number lines with increasingly large numbers to count on from the smaller number to the larger number.

\[
\begin{align*}
94 - 56 &= 30 + 4 + 4 = 38 \\
\end{align*}
\]

\[
\begin{align*}
+30 &+4 \quad +4 \\
56 &86 \quad 90 \quad 94 \\
\end{align*}
\]

Counting up to find the difference

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

\[
\begin{align*}
112 - 99 &= 13 \\
\end{align*}
\]

\[
\begin{align*}
+1 &+10 \quad +2 \\
99 &100 \quad 110 \quad 112 \\
\end{align*}
\]
### Key Stage 2

At this stage, children need to continue to develop mental methods for subtraction and begin to use formal methods for columnar subtraction. During year 3 they should be using numbers up to 3 digits, year 4 up to 4 digits and during year 5 4 digits and above.

#### Partitioning without exchanging

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>89 - 57 = (80 - 50) + (9 - 7)</td>
<td></td>
</tr>
<tr>
<td>= 30 + 2</td>
<td></td>
</tr>
<tr>
<td>= 32</td>
<td></td>
</tr>
</tbody>
</table>

#### Partitioning the second number only

Partition only second number when the ones in the largest number is smaller than the ones in the smaller number

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>92 - 36 = (92 - 30) - 6</td>
<td></td>
</tr>
<tr>
<td>= 62 - 6</td>
<td></td>
</tr>
<tr>
<td>= 56</td>
<td></td>
</tr>
</tbody>
</table>

**6 (ones in the second number) is larger than the ones in the first number**

#### Expanded decomposition without exchanging

Partitioning both numbers into tens and ones where ones are placed under ones and tens under tens

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>89 - 57 = 80 + 9</td>
<td></td>
</tr>
<tr>
<td>50 + 7</td>
<td></td>
</tr>
<tr>
<td>30 + 2 = 32</td>
<td></td>
</tr>
</tbody>
</table>

#### Expanded decomposition with exchanging

1 can not be taken away from 6, therefore exchange is necessary.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>71 - 46 =</td>
<td></td>
</tr>
<tr>
<td><strong>Step 1</strong></td>
<td></td>
</tr>
<tr>
<td>70 and 1</td>
<td></td>
</tr>
<tr>
<td>- 40 and 6</td>
<td></td>
</tr>
<tr>
<td><strong>Step 2</strong></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
</tr>
<tr>
<td>70 and 11</td>
<td></td>
</tr>
<tr>
<td>- 40 and 6</td>
<td></td>
</tr>
<tr>
<td>20 and 5 = 2</td>
<td></td>
</tr>
</tbody>
</table>

Exchange one ten and regroup it as 10 ones, therefore 11 – 6 can be done.
Children will use expanded decomposition with increasingly large numbers

$$754 - 86 =$$

**Step 1**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>50</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>80</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>__________</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step:** This would be recorded by the children as:

- Adjust from hundreds to tens
- Adjust from tens to ones

$$600 \quad 140$$

$$700 \quad 50 \quad 14$$

- 80

$$600 \quad 60 \quad 8 = 668$$

Compact decomposition without exchanging

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>T</td>
<td>U</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

They will move onto compact decomposition when ready

Compact decomposition with exchanging

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>H T U</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7 8</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>8</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>6 6</td>
<td>8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 4 can not be taken away from 6, therefore a ten needs to be carried over.
- 4 can not be taken away from 8, therefore one hundred needs to be carried over.

✓ Children will use compact decomposition with increasingly large numbers
### Expanded Decomposition of Decimals without Exchanging

\[
\begin{align*}
\£8.85 &= 8 \text{ and } 0.8 \text{ and } 0.05 \\
-\£4.32 &= -4 \text{ and } 0.3 \text{ and } 0.02 \\
\hline
4 \text{ and } 0.5 \text{ and } 0.03 &= \£4.57
\end{align*}
\]

### Compact Decomposition

- **Example:**
  
  | 5 1 3 1 
  | --- 
  | 6 4 6 7 
  | --- 
  | - 2 6 8 4 
  | --- 
  | 3 7 8 3 

  3 cannot be taken away from 6, therefore exchange is necessary.

  6 cannot be taken away from 8, therefore exchange is necessary.

### Expanded Decomposition of Decimals with Exchanging

- **Example:**
  
  \[
  \begin{align*}
  \£8.95 &= 8 \text{ and } 0.9 \text{ and } 0.05 \\
  -\£4.38 &= -4 \text{ and } 0.3 \text{ and } 0.08 \\
  \hline
  4 \text{ and } 0.5 \text{ and } 0.07 &= \£4.57
  \end{align*}
  \]

### Compact Decomposition with Exchanging

\[
\begin{align*}
\£8.85 &
\end{align*}
\]

\[
\begin{align*}
\£4.38 \\
\£4.57
\end{align*}
\]
Progression in Written Calculations:

MULTIPLICATION

Key Vocabulary:

lots of  groups of  times  multiply  multiplication multiple  product  double  repeated addition once, twice, three times  array  row  column

Using multiplication facts:

It is essential that pupils have a quick recall of the times tables facts. Below is a guide to show what facts they should learn and when.

Year 1
Count in multiples of 2s, 5s and 10s

Year 2
Recall multiplication and division facts for the 2, 5 and 10 multiplication tables

Year 3
Recall multiplication and division facts for the 3, 4 and 8 multiplication tables

Year 4
Derive and recall quickly all multiplication and division facts up to 12 x 12.
**EARLY STAGE 1:**

- The focus in the early stage is on addition and addition strategies so the links to multiplication are limited.
- Much of it comes in the form of repeated addition of small groups of numbers.
- At this stage the children will still be doing a lot of concrete work.
- Some children may extend on their experiences and represent in pictorial form.

<table>
<thead>
<tr>
<th>Practical activities</th>
<th>Pictorial representations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 plus 2 plus 2 makes 6</td>
</tr>
</tbody>
</table>

**Key Stage 1**

At this stage children should have lots of experience of counting in 2s, 3s, 5s and 10s. They should use repeated addition, concrete materials, mental methods and multiplication and division facts to enable them to solve problems. They should be introduced to the x and = symbols.

<table>
<thead>
<tr>
<th>Groups/lots of 2’s and 10’s, later in 5’s using pictorial representations</th>
<th>2 groups of 5 makes 10 or 2 lots of 5 makes 20</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Repeated addition</th>
<th>Know that 2 x 4 is the same as: 2 + 2 + 2 + 2 or 4 groups of 2 or 4 lots of 2</th>
</tr>
</thead>
</table>

Children given 10 candles and asked to make 2 equal groups

Children are introduced to the x sign and can start using it to record practical activities
Count forwards and backwards in 2s, 5s and 10s. (Yr2 – count in 3s)

Continue using repeated addition, however with larger numbers

Begin using a number line to show repeated addition

**Commutativity**

Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on a prepared number line or arrays.
Children should be able to model a multiplication calculation using an array. Using the language of rows and columns.

**Arrays**

- $5 \times 3 = 15$
- $3 \times 5 = 15$

Repeated addition

Children use blank number lines to support their understanding.

- $0, 6, 12, 18, 24$
- $6 + 6 + 6 + 6 = 24$

**Children know that:**

- $4 \times 6$ is
- $6 + 6 + 6 + 6$ or
- $4$ lots of $6$ or
- $6 \times 4$

Continue using arrays with larger numbers

- $9 \times 4 = 36$
- $4 \times 9 = 36$

Partitioning horizontally, multiplying the tens first

- $38 \times 5 = (30 \times 5) + (8 \times 5)$
- $= 150 + 40$
- $= 190$

Use known fact:

- $3 \times 5$
Key Stage 2
At this stage children move on to using formal methods of multiplication moving from the grid method to columnar methods including long multiplication. Year 3 should be using x facts that they are familiar with, year 4 should be multiplying 2 and 3 digit numbers by a 1 digit number, year 5 should be multiplying up to 4 digit numbers by 2 and 1 digit numbers (including long multiplication for 2 digit numbers 23 x 25) and year 6 should be able to multiply numbers up to 4 digits by a 2 digit number using the formal method of long multiplication.

Grid method for short multiplication (multiplication by a single digit)

<table>
<thead>
<tr>
<th>x</th>
<th>20</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>160</td>
<td>24</td>
</tr>
</tbody>
</table>

= 160 + 24 = 184

153 x 4

<table>
<thead>
<tr>
<th>x</th>
<th>100</th>
<th>50</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>400</td>
<td>200</td>
<td>12</td>
</tr>
</tbody>
</table>

= 400 + 200 + 12 = 612

Extending to a 3-digit number multiplied by a single digit number

<table>
<thead>
<tr>
<th>x</th>
<th>100</th>
<th>50</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>2300</td>
<td>230</td>
<td>9</td>
</tr>
</tbody>
</table>

Children partition vertically to lead to a more formal recording. Only when they are ready.

STAGE 5:

Grid method for short multiplication (multiplication by a single digit)

<table>
<thead>
<tr>
<th>x</th>
<th>300</th>
<th>40</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>2700</td>
<td>360</td>
<td>54</td>
</tr>
</tbody>
</table>

2700 + 360 + 54 = 3114

Multiplying a 2-digit number by a single digit number

<table>
<thead>
<tr>
<th>x</th>
<th>100</th>
<th>50</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>161</td>
<td>1610</td>
<td>161</td>
<td>161</td>
</tr>
</tbody>
</table>

23 x 8

Grid method for short multiplication (multiplication by a single digit)
## Grid method for long multiplication (multiplication by a 2-digit)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>70</td>
<td>2</td>
</tr>
<tr>
<td>30</td>
<td>2100</td>
<td>60</td>
</tr>
<tr>
<td>8</td>
<td>560</td>
<td>16</td>
</tr>
</tbody>
</table>

\[
72 \times 38 = \begin{array}{c}
2100 \\
+ 560 \\
+ 60 \\
+ 16 \\
\hline
2736
\end{array}
\]

Multiplying decimals using the grid method

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

\[
4.9 \times 3 = \begin{array}{c}
+ 2.7 \\
\hline
14.7
\end{array}
\]

## Formal written multiplication – expanded form

\[
423 \times 7 = \begin{array}{c}
21 \times (3 \times 7) \\
140 \times (20 \times 7) \\
2800 \times (400 \times 7) \\
\hline
2961
\end{array}
\]

## Grid method for short multiplication ThHTU x U

<table>
<thead>
<tr>
<th></th>
<th>4000</th>
<th>300</th>
<th>40</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>32000</td>
<td>2400</td>
<td>320</td>
<td>48</td>
</tr>
</tbody>
</table>

\[
4346 \times 8 = \begin{array}{c}
32000 \\
+ 2400 \\
+ 320 \\
+ 48 \\
\hline
34768
\end{array}
\]

## Grid method for long multiplication HTU x TU

<table>
<thead>
<tr>
<th></th>
<th>300</th>
<th>70</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>6000</td>
<td>1400</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>1200</td>
<td>280</td>
<td>8</td>
</tr>
</tbody>
</table>

\[
372 \times 24 = \begin{array}{c}
6000 \\
+ 1400 \\
+ 1200 \\
+ 280 \\
+ 40 \\
+ 8 \\
\hline
8928
\end{array}
\]

\[
\text{1}
\]
### Multiplying decimals using the grid method

<table>
<thead>
<tr>
<th>x</th>
<th>4</th>
<th>0.9</th>
<th>0.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>12</td>
<td>2.7</td>
<td>0.06</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c}
4.92 \\
\times 3
\end{array}
\]

\[
\begin{array}{c}
12.76
\end{array}
\]

Moving onto numbers with 2 decimal places when ready

### Expanded form of short multiplication, moving onto the compact method once secure

**Short Multiplication:**

\[
\begin{array}{c}
4346 \\
\times 8
\end{array}
\]

\[
\begin{array}{c}
34768
\end{array}
\]

Leading to 234

### Expanded form of long multiplication, moving onto the compact method once secure

**Long Multiplication:**

\[
\begin{array}{c}
352 \\
\times 27
\end{array}
\]

\[
\begin{array}{c}
9504
\end{array}
\]

Moving onto numbers with 2 decimal places when ready
Progression in Written Calculations:

**DIVISION**

Key Vocabulary:

<table>
<thead>
<tr>
<th>lots of</th>
<th>groups of</th>
<th>group</th>
<th>share</th>
</tr>
</thead>
<tbody>
<tr>
<td>shared between</td>
<td>divide</td>
<td>divide into</td>
<td></td>
</tr>
<tr>
<td>division</td>
<td>divided by</td>
<td>remainder</td>
<td></td>
</tr>
<tr>
<td>factor</td>
<td>quotient</td>
<td>divisible</td>
<td></td>
</tr>
</tbody>
</table>

**EARLY STAGE 1:**

At the early stage, the focus is primarily on practical experiences involving sharing items into equal groups with the use of concrete apparatus for the children to physically count and see.

<table>
<thead>
<tr>
<th>Practical activities</th>
<th>Pictorial representations</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 shared into 3 equal groups is 2</td>
<td></td>
</tr>
</tbody>
</table>

*Understand division as sharing and grouping*
**Key Stage 1**

Emphasis in this stage is in counting in steps and the recall and use of multiplication facts. Children should be introduced to the division ÷ symbol and solve division problems using concrete objects, arrays, repeated subtraction and division facts.

Sharing equally
Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s and 3s.

Count forwards and backwards in 2s/5s/10s (3s in year 2 from any number).

- Introduced to the ÷ sign
- But do not use to record yet.

There are 6 strawberries. How many people can have 2 each? (How many 2s make 6?)

**Grouping – There are 6 sweets. How many people can have 2 each? (How many 2's make 6?)**

There are 6 strawberries, how many people can have 2 strawberries each?

**OR**

There are 6 strawberries, how many strawberries does each person get if there are 3 people.

Repeated subtraction using a prepared number

\[ 12 \div 3 = 4 \]

- 4th Group
- 3rd group
- 2nd group
- 1st group
as $10 \div 5$ as ‘how many 5s make 10?’

Emphasis on grouping rather than sharing using a blank number line

Repeated subtraction using a number line
Children use blank number lines to support their understanding

Introducing remainders
Using a number line to support understanding

$24 \div 6 = 4$

$27 \div 6 = 4 \text{ r } 3$
**Key Stage 2**

At this stage children should start to use formal written methods for division, moving from the chunking method, to short division and long division. By year 5 children should dividing numbers up to 4 digits by a 1 digit using the short division method. In year 6 they should be dividing numbers up to 4 digit by 2 digit whole numbers using short or long division. At this stage remainders need to be interpreted as either whole number remainders, fractions or rounding – according to context.

### Chunking method:

2 digit number divided by a single digit number

Children begin with familiar multiples 2, 5, 10 and 3

<table>
<thead>
<tr>
<th>72 ÷ 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ) 72</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>42</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

Answer: 10 + 10 + 2 + 2 = 24

Follows the repeated subtraction method. Children take-away small chunks at a time until they get to zero.

### Leading to subtraction of other multiples

96 ÷ 6

<table>
<thead>
<tr>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 ) 96</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

Answer: 10 + 6 = 16

### Continue vertical written method (chunking) with and without remainders

Subtracting larger multiples

<table>
<thead>
<tr>
<th>196 ÷ 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 r 4</td>
</tr>
<tr>
<td>6 ) 196</td>
</tr>
<tr>
<td>180</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

Answer: 30 + 2 = 32 with 4 left over 32 r 4

Children become more efficient and will now look for opportunities to reduce workings by subtracting larger multiples. e.g. Instead of subtracting 10 x 6 three times, children will subtract 30 x 6 and use known facts to assist, i.e: 3 x 6
Continue subtracting larger multiples

Continue chunking method to solve 3 digit numbers divided by a single digit number, extending to 3 digit numbers divided by 2-digit numbers

\[
\begin{array}{c}
972 \div 36 \\
36 \overline{) 972} \\
- 720 \\
252 \\
- 252 \\
0
\end{array}
\]

Using known facts: \( 2 \times 26 \) or double 36 = 72, adding the zero to make 720

\[
\begin{align*}
20 \times 36 & = 20 \times 26 + 4 \times 36 = 20 \times 26 + 7 \times 36 \\
& = 20 \times 26 + 7 \times 36 \\
\end{align*}
\]

Answer: \( 20 + 7 = 27 \)

<table>
<thead>
<tr>
<th>Short division</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 6 ) ( 196 )</td>
</tr>
<tr>
<td>( \overline{3 \ 2 \ r4} )</td>
</tr>
<tr>
<td>Step 1: 6 into 19 = 3 times with 1 remaining. The 1 is carried over to make 16.</td>
</tr>
<tr>
<td>Step 2: 6 into 16 = 2 times with 4 remaining</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Remainders as fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Step 1:} \ 6 \text{ into } 19 = 3 \text{ times with } 1 \text{ remaining. The } 1 \text{ is carried over to make } 16. )</td>
</tr>
<tr>
<td>( \text{Step 2:} \ 6 \text{ into } 16 = 2 \text{ times with } 4 \text{ remaining. As a fraction } 4/6 )</td>
</tr>
</tbody>
</table>
| Long division – with remainders | \[ \begin{array}{c}
432 \div 15 \text{ becomes} \\
\begin{array}{c|ccc}
6 & 8 & r & 12 \\
\hline
1 & 5 & \overline{4} & 3 & 2 \\
3 & 0 & 0 & \quad \\
1 & 3 & 2 & \quad \\
1 & 2 & 0 & \quad \\
\hline
1 & 2 & \\
\end{array}
\end{array} \\
\text{Answer: 28 remainder 12} |
| --- | --- |
| Long division – with fraction as remainder | \[ \begin{array}{c}
2 & 8 \\
1 & 5 \overline{4} & 3 & 2 \\
3 & 0 & 0 & \quad 15 \times 20 \\
1 & 3 & 2 & \quad 15 \times 8 \\
1 & 2 & 0 & \\
\hline
1 & 2 & \\
\end{array} \\
\frac{12}{15} = \frac{4}{5} \\
\text{Answer: } 28 \frac{4}{5} |
| Long division with decimal remainders | \[ \begin{array}{c}
2 & 8 \cdot 8 \\
1 & 5 \overline{4} & 3 & 2 \cdot 0 \\
3 & 0 \downarrow \\
1 & 3 & 2 \\
1 & 2 & 0 \downarrow \\
1 & 2 & 0 \\
1 & 2 & 0 \\
\hline
0 & \\
\end{array} \\
\text{Answer: } 28.8 |