

Progression in Written Calculations:


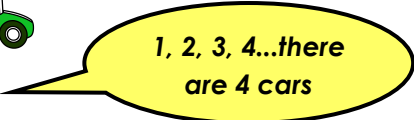

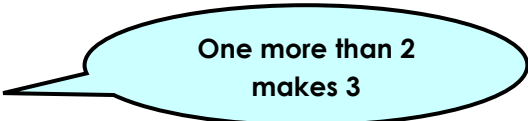

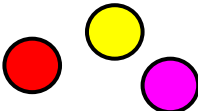
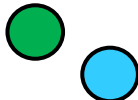
ADDITION

Key Vocabulary:

add addition and plus count on more
increase makes sum total altogether

EARLY STAGE :

At this stage, children are not expected to use paper and pencil procedures for addition. Their experience of these operations will be a mixture of practical, oral and mental work.

Recognise numbers 0 to 10	0 1 2 3 4 5 6 7 8 9 10					
Count reliably up to 10 everyday objects	 					
Count one more than a number	  					
Begin to add two small groups of objects together	 and  makes 5					
Count in ones to add numbers together	3 add 2 makes 5 <table border="1"><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr></table>	1	2	3	4	5
1	2	3	4	5		

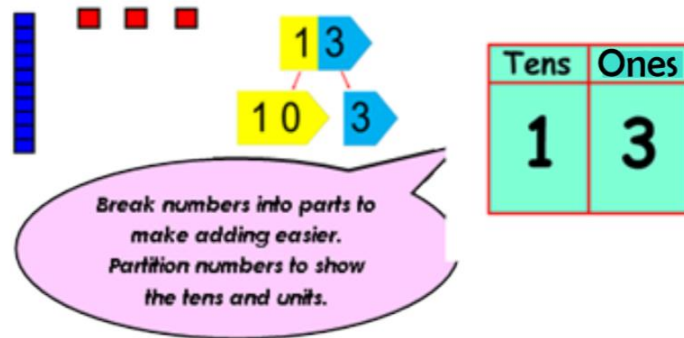
Before pupils can move to recording $3+2$, they will need experience of practical addition, and an ability to respond to mathematical vocabulary practically. Children use signs and symbols to record practical work.

Key Stage 1

At this stage, pupils need experience of adding using concrete objects and pictorial representations in a wide variety of contexts. Pupils will use number lines extensively to develop mental calculations which will assist them when recording. Children begin to use blank number lines to support their own calculations.

Begin to read and use the + and = signs to record practical work and mental calculations in a number sentence	<div><div><div>●●●</div><div>+</div><div>●●</div><div>=</div><div>5</div></div><div><div>3</div><div>+</div><div>2</div><div>=</div><div>5</div></div></div>																																																																																																				
Use concrete objects and pictorial representations to assist in adding one and two digit numbers	<div><div><div>1</div><div>+</div><div>2</div><div>=</div><div>3</div></div><div><div>🧸</div><div>🧸🧸</div><div>🧸🧸🧸</div></div></div>																																																																																																				
Children begin to use prepared number lines to support their own calculations to add one and two digit numbers	<div><div>Counting up in 1's</div><div>8 + 5 = 13</div><div><div>+1 +1 +1 +1 +1</div><div>0 1 2 3 4 5 6 7 8 9 10 11 12 13</div></div></div>																																																																																																				
Know by heart all number bonds with a total of 10 or 20.	<div><div>100</div><div>91</div><div>82</div><div>73</div><div>64</div><div>55</div><div>46</div><div>37</div><div>28</div><div>19</div><div>0</div><div>1</div><div>2</div><div>3</div></div>																																																																																																				
Counting on in twos, fives and tens.	<div>Counting in 2s</div> <table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr><tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr><tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr><tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr><tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr><tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr><tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr><tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr><tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr></table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10																																																																																												
11	12	13	14	15	16	17	18	19	20																																																																																												
21	22	23	24	25	26	27	28	29	30																																																																																												
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91	92	93	94	95	96	97	98	99	100																																																																																												

Partition 2-digit numbers in order to add at a later stage



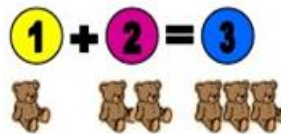
Recall all number bonds with a total of 10 and 20 fluently and use this to use related number bonds to 100.



Because $8 + 2 = 10$
I know that
 $80 + 20 = 100$

Continue to add and subtract numbers using concrete objects and pictorial representations to add:

- a 2-digit number and ones
- a 2-digit number and tens
- two 2-digit numbers
- adding three 1-digit numbers

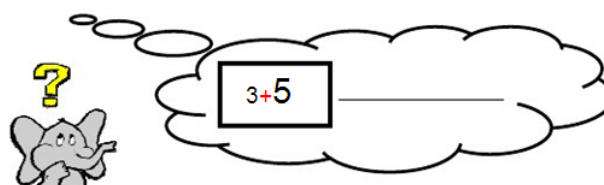


Know that addition can be done in any order.

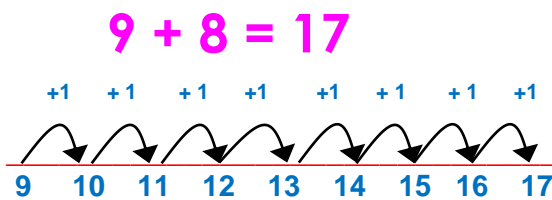


$1 + 2 = 3$
is the
same as
 $2 + 1 = 3$

To put the biggest number first when counting on.

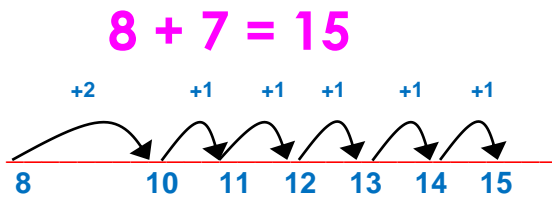


Add two single-digit numbers, counting up in ones



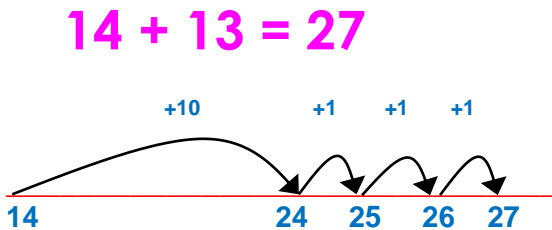
Continue
Counting up in
1's

Add two single-digit numbers that bridge 10

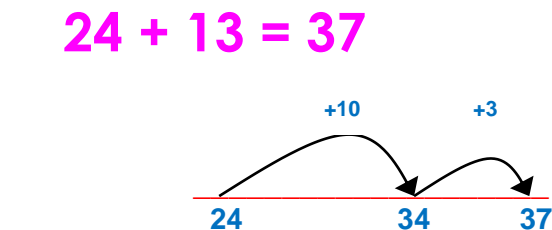


Using
knowledge of
number bonds
to 10 ($8+2=10$)

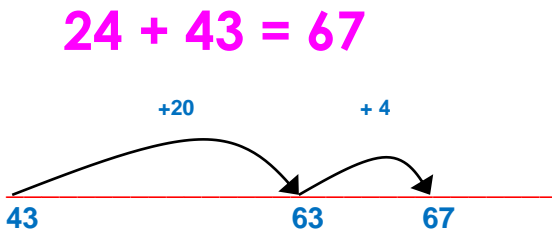
Counting on in tens and ones



Adding the tens and ones in one jump

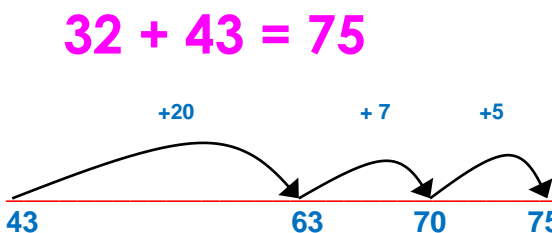


Adding the tens in one jump and the ones in one jump



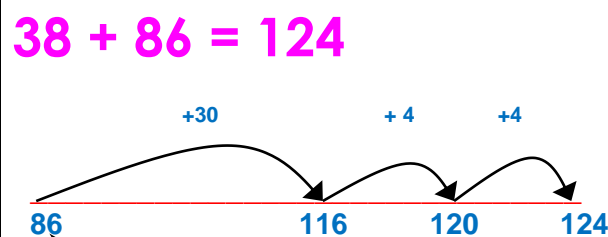
Remember to
start from the
biggest number

Add two 2-digit numbers that bridge 10



Using knowledge
of number bonds
to 10 ($3+7=10$)

Children will continue to use empty number lines with increasingly large numbers, and counting on from the largest number irrespective of the order of the calculation



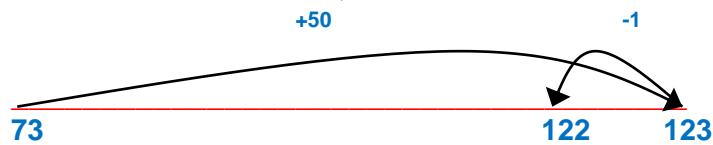
Start from the
biggest number

Using knowledge
of number bonds
to 10 ($6+4=10$)

Children will continue to use empty number lines with increasingly large numbers, including **compensation** where appropriate

$$49 + 73 = 122$$

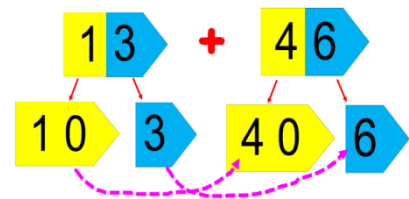
Round 49 to the nearest ten (50) then take-away the extra 1 that was added



Horizontal partitioning

Begin to record mental calculations using partitioning, splitting the tens and ones, and recombining

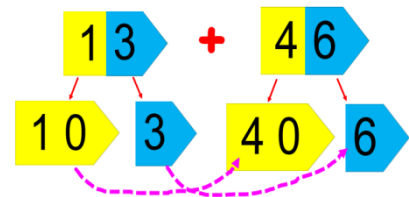
$$\begin{aligned} 13 + 46 &= (10 + 40) + (3 + 6) \\ &= 50 + 9 \\ &= 59 \end{aligned}$$



Column partitioning

Partitioning both numbers into tens and ones where ones are placed under ones and tens under tens to prepare children for formal columnar methods.

$$\begin{aligned} 13 + 46 &= 10 + 3 \\ &\quad \underline{40 + 6} \\ &\quad \underline{50 + 9} = 59 \end{aligned}$$



Key Stage 2

During Key Stage 2, children should become increasingly confident using the formal columnar method of addition. In Year Three they should be using 3 digit numbers, in Year 4 they should be using 4 digit numbers and in Year Five they should be able to add numbers with more than 4 digits. By Year Six this method should be completely embedded. Children should also be able to add decimals in the context of measure (money, length etc).

Expanded column method adding the ones first, then tens, then the hundreds	<div><div>67</div><div><div>+43</div><div>10 (7+3)</div><div>100 (60+40)</div><div>110</div></div></div> <div>Record mental calculation</div> <div><div>126</div><div><div>+54</div><div>10 (6+4)</div><div>70 (20+50)</div><div>100 (100+0)</div><div>180</div></div></div>								
Use expanded column method when adding money, beginning with decimals that require no carrying and then move onto carrying the tenths or hundredths only	<div><div>£ 2. 50</div><div><div>+£ 1. 75</div><div>5 (5p + 70p)</div><div>£ 1. 20 (50p + 70p)</div><div>£ 3. 00 (£2.00 + £1.00)</div><div>£ 4. 25</div></div></div> <div><table><tr><th>Tens</th><th>Ones</th><th>Tenths</th><th>Hundredths</th></tr><tr><td>1</td><td>7</td><td>6</td><td>3</td></tr></table></div>	Tens	Ones	Tenths	Hundredths	1	7	6	3
Tens	Ones	Tenths	Hundredths						
1	7	6	3						
Compact column method without carrying Children add from least significant number without carrying	<div><div>HTU</div><div><div>↓ ↓ ↓</div><div>3 4 2</div><div>+ 5 7</div><div>3 9 9</div></div></div> <div>Add the ones first, then the tens and lastly, the hundreds</div>								
Compact column method, carrying below the line (ONES only)	<div><div>HTU</div><div><div>↓ ↓ ↓</div><div>6 2 5</div><div>+ 4 8</div><div>6 7 3</div><div>1</div></div></div> <div>Children will begin with carrying only the ones below the line. 5+8 = 13. Record the 3 in the ones column, carrying the 1 ten.</div>								

<p>Compact column method, carrying below the line (TENS only)</p>	<div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: center;"> <p>H T U</p> <p>↓ ↓ ↓</p> $\begin{array}{r} 625 \\ + 94 \\ \hline 719 \\ 1 \end{array}$ </div> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; margin-left: 20px; background-color: #f0e6ff;"> <p>Children carry only the tens below the line. 2 tens + 9 tens = 11 tens. Record 1 ten in the tens column, carrying the 10 tens.</p> </div> </div>
<p>Compact column method, carrying ones, tens and hundreds</p>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> $\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ 11 \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ 111 \end{array}$ </div> </div>
<p>Carrying decimals, begin with decimals to 1 decimal place and then moving to 2 decimal places</p>	<div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; margin-right: 20px; background-color: #d1c4e9;"> <p>Carrying tenths only</p> </div> <div style="text-align: center;"> $\begin{array}{r} 32.5 \text{ km} \\ + 54.6 \text{ km} \\ \hline 87.1 \text{ km} \\ 1 \end{array}$ </div> </div>
<p>Carrying of both tenths and hundredths</p>	<div style="text-align: center;"> $\begin{array}{r} £6.72 \\ £8.56 \\ + £2.33 \\ \hline £17.61 \\ 1 \quad 1 \end{array}$ </div>
<p>Compact column method, carrying ones, tens, hundreds and thousands</p> <p>Adding more than two numbers</p>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> $\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ 111 \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ 111 \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ 121 \end{array}$ </div> </div>
<p>Add decimals with different numbers of places and different numbers of digits</p>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> $\begin{array}{r} 124.9 \\ 86.3 \\ \hline 211.2 \\ 111 \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 124.9 \\ 117.25 \\ \hline 242.15 \\ 11 \end{array}$ </div> </div>



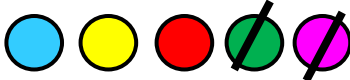
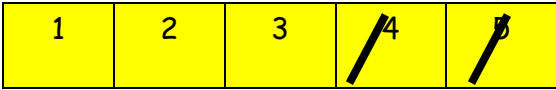
SUBTRACTION

Key Vocabulary:

take-away **count back** **subtract** **less**
fewer **minus** **difference** **difference between**



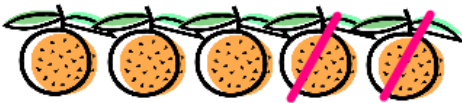


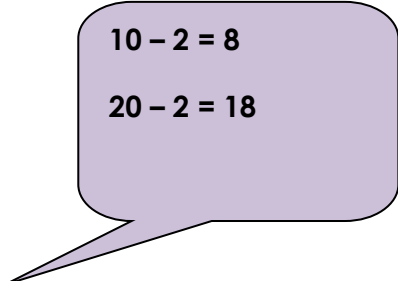
EARLY STAGE 1:

At this stage, children are not expected to use paper and pencil procedures for subtraction. Their experience of these operations will be a mixture of practical, oral and mental work.

Recognise numbers 0 to 10	0 1 2 3 4 5 6 7 8 9 10
Count back from 10	10 9 8 7 6 5 4 3 2 1 0
Count one less than a number	<div></div> <div></div> <div>1 less than 3 is 2</div>
Begin to subtract one small group from another	<div></div> <div></div> <div>5 take-away 2 leaves 3</div> <div>Using pictures or number tracks to subtract one group from another and counting what is left over.</div>

KEY STAGE 1

Before pupils can move to recording 5-2, they will need experience of practical subtraction, and an ability to respond to mathematical vocabulary practically. Children use signs and symbols to record practical work only when they are ready. At this stage, pupils use number lines extensively to develop mental calculations which will assist them when recording. Children begin to use blank number lines to support their own calculations.

Begin to use the - and = signs to record practical work and mental calculations in a number sentence	 leaves 3 $5 - 2 = 3$
Children begin to understand that subtraction is not commutative	$6 - 2 = 4$ $2 - 6 = 4$ ✗ 
Subtract 1 and 2 digit numbers to 20 – including 0.	$12 - 4 = 8$ $15 - 5 = 10$
Relationship to addition: Use models and images to show the inverse relationship between addition and subtraction. Use this to solve missing number problems.	 $5 - 2 = 3$  $3 + 2 = 5$
Recall subtraction facts to 20	 

Continue to add and subtract numbers using concrete objects and pictorial representations to add:

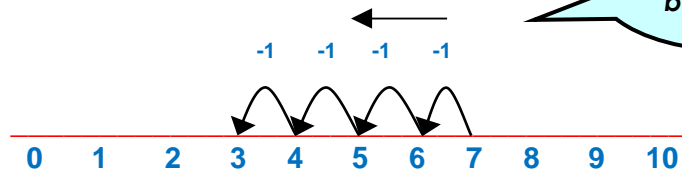
- a 2-digit number and ones
- a 2-digit number and tens
- two 2-digit numbers



$$3 - 2 = 1$$

Children begin to use **prepared** number lines to support their own calculations, beginning with counting back in ones

$$7 - 4 = 3$$



Counting back in 1's

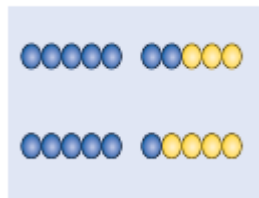
Using a **prepared** number line to find the difference by counting up



$$7 - 4 = 3$$

The numberline is used to show that $7 - 4$ means the 'difference between 7 and 4' or 'the difference between 4 and 7' and how many jumps they are apart.

Take – away numbers 0 - 20 from 20. Use these to use related facts up to 100.



$$10 - 3 = 7$$

$$100 - 30 = 70$$

Because I know:

$10 - 3 = 7$ I know:

$100 - 30 = 70$

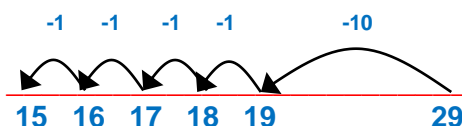
Count forwards and backwards in 1's, 2's, 5's and 10's

Counting in 1s										Counting in 2s									
Count forward →					Count backward ←														
1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100	91	92	93	94	95	96	97	98	99	100

Counting in 5s										Counting in 10s									
1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100	91	92	93	94	95	96	97	98	99	100

Empty number lines to count back, first counting back in tens and ones

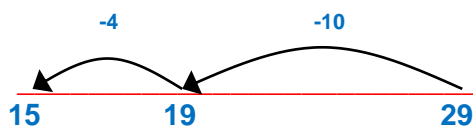
$$29 - 14 = 15$$



Start from left and count back

Empty number lines to count back, counting back in tens and the ones in one jump

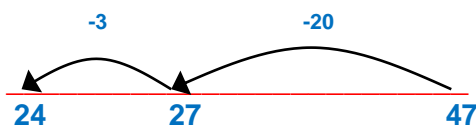
$$29 - 14 = 15$$



Using the known fact $9 - 4 = 5$ to count back in one jump

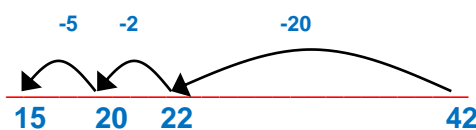
Empty number lines to count back, counting back in tens and the ones in one jump

$$47 - 23 = 24$$



Subtract two 2-digit numbers that bridge 10

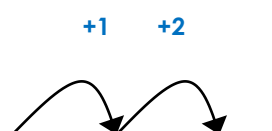
$$42 - 27 = 15$$



Using knowledge of number bonds to 10 ($7+3=10$) ($10-3=7$)

Empty number lines to count up

$$42 - 39 = 3$$



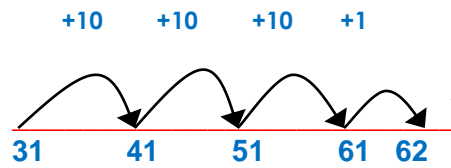
It can be more efficient to count on if the numbers involved

Calculations close together

in the calculation are close together or if the numbers involved in the calculation are near to multiples of 10, 100 etc

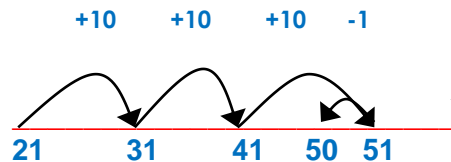
39 40 42

$$62 - 31 = 31$$



Near to multiples of 10, 100 etc

$$50 - 21 = 29$$



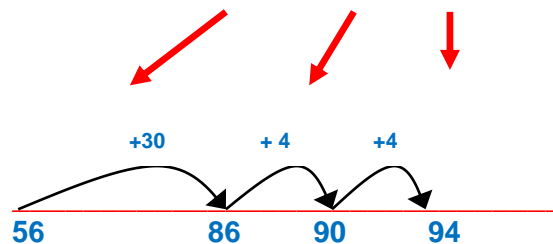
Subtracting 9 or 11, 19 or 21 etc

Making decisions about counting up or back

With practice, children will need to decide whether to count back or forward and which method is more efficient to use.

Empty number lines with large numbers to count on
Children will continue to use empty number lines with increasingly large numbers to count on from the smaller number to the larger number

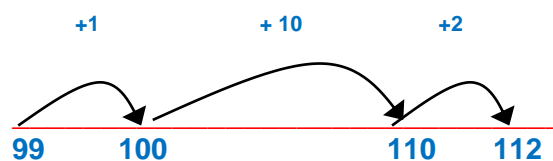
$$94 - 56 = 30 + 4 + 4 = 38$$

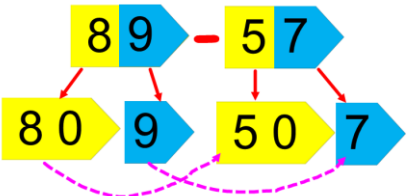
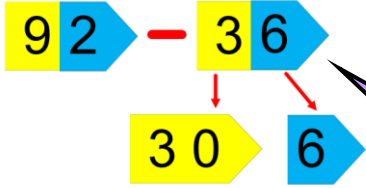


Counting up to find the difference

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used

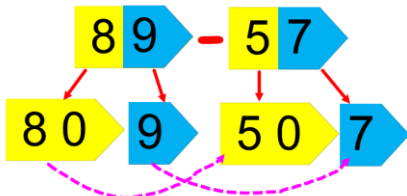
$$112 - 99 = 13$$



<p><u>Partitioning without exchanging</u></p> <p><u>Partition both tens and ones</u></p>	$89 - 57 = (80 - 50) + (9 - 7)$ $= 30 + 2$ $= 32$ 
<p><u>Partitioning the second number only</u></p> <p>Partition only second number when the ones in the largest number is smaller than the ones in the smaller number</p>	 $92 - 36 = (92 - 30) - 6$ $= 62 - 6$ $= 56$ <div data-bbox="879 651 1433 824" style="border: 1px solid black; border-radius: 50%; padding: 10px; background-color: #f0f0f0;"> <p>6 (ones in the second number) is larger than the ones in the first number</p> </div>

Key Stage 2

At this stage, children need to continue to develop mental methods for subtraction and begin to use formal methods for columnar subtraction. During year 3 they should be using numbers up to 3 digits, year 4 up to 4 digits and during year 5 4 digits and above.

<p>Expanded decomposition without exchanging</p> <p>Partitioning both numbers into tens and ones where ones are placed under ones and tens under tens</p>	$89 - 57 = 80 + 9$ $\underline{50 + 7}$ $\underline{30 + 2} = 32$ 
<p>Expanded decomposition with exchanging</p>	$71 - 46 =$ <p><u>Step 1</u></p> $\begin{array}{r} 70 \text{ and } 1 \\ - 40 \text{ and } 6 \\ \hline \end{array}$ <p><u>Step 2</u></p> $\begin{array}{r} 60 \\ \cancel{70} \text{ and } 11 \\ - 40 \text{ and } 6 \\ \hline 20 \text{ and } 5 = 2 \end{array}$ <div data-bbox="1082 1570 1437 1787" style="border: 1px solid black; padding: 5px; background-color: #e0ffff;"> <p>1 can not be taken away from 6, therefore exchange is necessary.</p> </div> <div data-bbox="1082 1868 1437 2085" style="border: 1px solid black; padding: 5px; background-color: #ffffe0;"> <p>Exchange one ten and regroup it as 10 ones, therefore 11 - 6 can be done.</p> </div>

Children will use expanded decomposition with increasingly large numbers

$$754 - 86 =$$

Step 1

$$\begin{array}{r} 700 \text{ and } 50 \text{ and } 4 \\ - \quad \quad \quad 80 \text{ and } 6 \\ \hline \end{array}$$

4 can not be taken away from 6, therefore exchange is necessary.

Step: This would be recorded by the children as:

adjust from
hundreds
to tens

adjust from
tens to ones

$$\begin{array}{r} 600 \quad \quad 140 \\ \cancel{700} \text{ and } \cancel{50} \text{ and } 14 \\ - \quad \quad \quad 80 \text{ and } 6 \\ \hline 600 \text{ and } 60 \text{ and } 8 = 668 \end{array}$$

Compact decomposition without exchanging

$$\begin{array}{r} \text{HTU} \\ \downarrow \downarrow \downarrow \\ 564 \\ - 321 \\ \hline 243 \end{array}$$

They will move onto compact decomposition when ready

Compact decomposition with exchanging

$$\begin{array}{r} \text{HTU} \\ \downarrow \downarrow \downarrow \\ \overset{6}{\cancel{7}} \overset{14}{\cancel{5}} \overset{1}{\cancel{4}} \\ - \quad 86 \\ \hline 668 \end{array}$$

4 can not be taken away from 6, therefore a ten needs to be carried over.

4 can not be taken away from 8, therefore one hundred needs to be carried over.

- ✓ Children will use compact decomposition with increasingly large numbers

Expanded decomposition of decimals without exchanging	$ \begin{array}{r} \text{£}8.85 = 8 \text{ and } 0.8 \text{ and } 0.05 \\ \text{£}4.32 = 4 \text{ and } 0.3 \text{ and } 0.02 \\ \hline 4 \text{ and } 0.5 \text{ and } 0.03 = \text{£}4.57 \end{array} $
---	--

Compact Decomposition be able to subtract numbers with different numbers of digits	<div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; padding: 5px; background-color: #f0f0f0; margin-right: 20px;"> 3 can not be taken away from 6, therefore exchange is necessary. </div> <div style="text-align: center;"> $\begin{array}{r} 5 \ 13 \ 1 \\ \cancel{6} \cancel{4} \cancel{6} 7 \\ - \quad 2684 \\ \hline 3783 \end{array}$ </div> <div style="border: 1px solid black; padding: 5px; background-color: #e0e0e0; margin-left: 20px;"> 6 can not be taken away from 8, therefore exchange is necessary. </div> </div>
--	---

Expanded decomposition of decimals WITH exchanging be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places	$ \begin{array}{r} \text{£}8.95 = 8 \text{ and } 0.9 \text{ and } 0.05 \\ \text{£}4.38 = 4 \text{ and } 0.3 \text{ and } 0.08 \\ \hline 4 \text{ and } 0.5 \text{ and } 0.07 \end{array} $
---	---

Compact decomposition of decimals WITH exchanging	$ \begin{array}{r} \cancel{7} \ \cancel{17} \ \cancel{1} \\ \text{£} \cancel{8} \cancel{.} \cancel{8} \cancel{5} \\ \text{£}4.38 \\ \hline \text{£}4.57 \end{array} $
---	---

MULTIPLICATION

Key Vocabulary:

lots of groups of times multiply multiplication
multiple product double repeated addition
once, twice, three times array row column

Using multiplication facts:

It is essential that pupils have a quick recall of the times tables facts. Below is a guide to show what facts they should learn and when.

Year 1
Count in
multiples of 2s,
5s and 10s

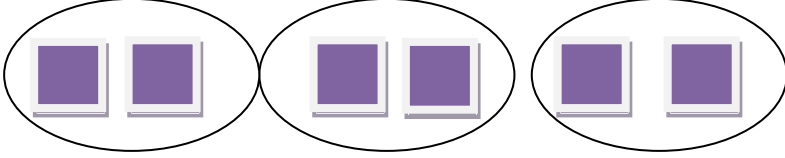
Year 2
Recall
multiplication and
division facts for the
2, 5 and 10
multiplication
tables

Year 3
Recall multiplication
and division facts for
the 3, 4 and 8
multiplication tables

Year 4
Derive and recall
quickly all
multiplication and
division facts up to 12
x 12.

EARLY STAGE 1:



- ✓ The focus in the early stage is on addition and addition strategies so the links to multiplication are limited.
- ✓ Much of it comes in the form of repeated addition of small groups of numbers.
- ✓ At this stage the children will still be doing a lot of concrete work.
- ✓ Some children may extend on their experiences and represent in pictorial form.

Practical activities	
Pictorial representations	

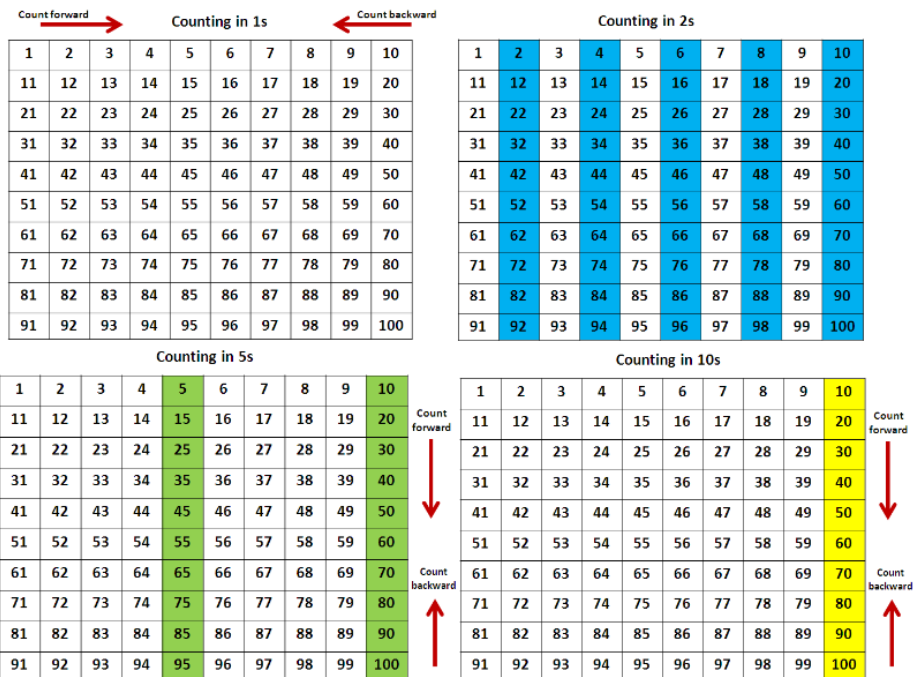
2 plus 2 plus 2 makes 6

Key Stage 1

At this stage children should have lots of experience of counting in 2s, 3s, 5s and 10s. They should use repeated addition, concrete materials, mental methods and multiplication and division facts to enable them to solve problems. They should be introduced to the x and = symbols.

Groups/lots of 2's and 10's, later in 5's using pictorial representations	 2 groups of 5 makes 10 or 2 lots of 5 makes 10 <i>Children given 10 candles and asked to make 2 equal groups</i>
Repeated addition	 2 + 2 + 2 + 2 <u>Know that 2 x 4 is the same as:</u> 2 + 2 + 2 + 2 or 4 groups of 2 or 4 lots of 2 <i>Children are introduced to the X sign and can start using it to record practical activities</i>

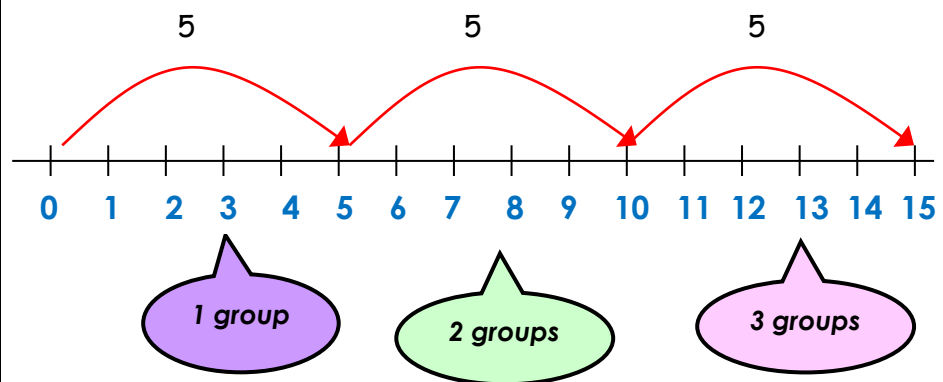
Count forwards and backwards in 2s, 5s and 10s. (Yr2 – count in 3s)



Continue using repeated addition, however with larger numbers

Begin using a number line to show repeated addition

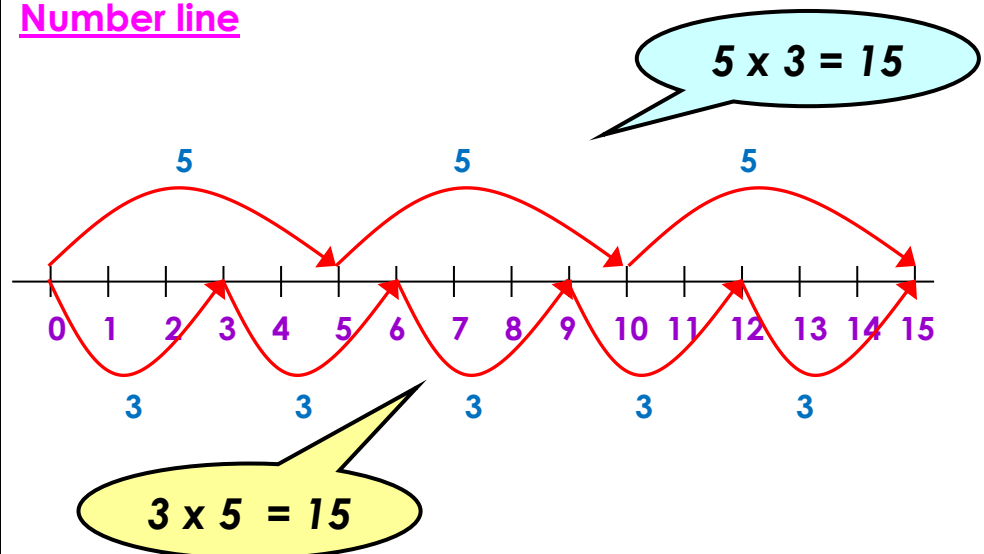
$$5 + 5 + 5 = 15$$



Commutativity

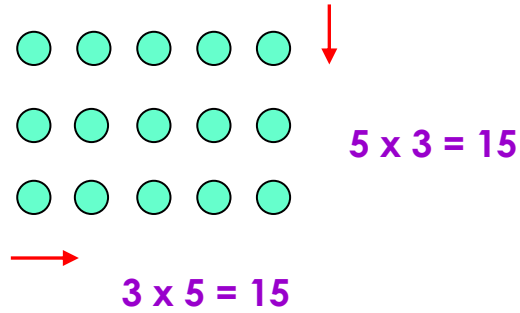
Children should know that 3×5 has the same answer as 5×3 . This can also be shown on a prepared number line or arrays

Number line



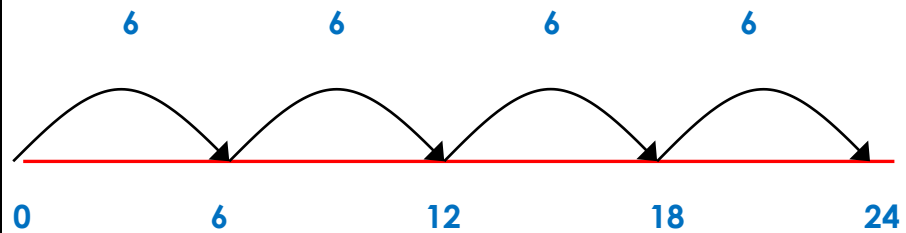
Children should be able to model a multiplication calculation using an array. Using the language of rows and columns.

Arrays



Repeated addition

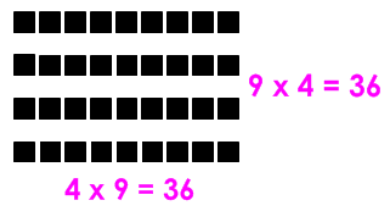
Children use blank number lines to support their understanding



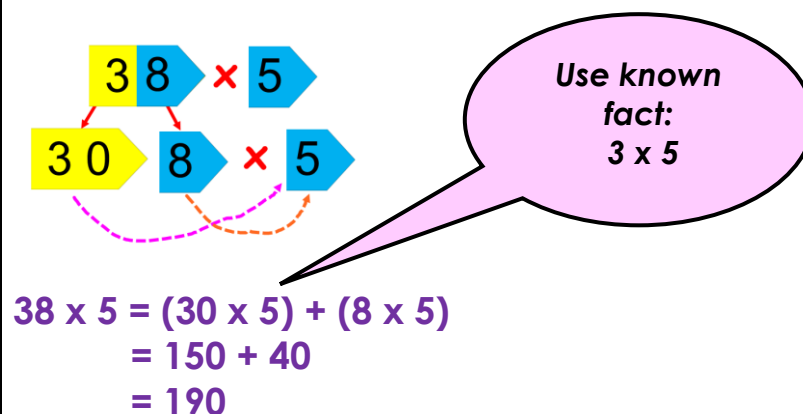
Children know that:

4×6 is
 $6 + 6 + 6 + 6$ or
 4 lots of 6 or
 6×4

Continue using arrays with larger numbers



Partitioning horizontally, multiplying the tens first



Key Stage 2

At this stage children move on to using formal methods of multiplication moving from the grid method to columnar methods including long multiplication. Year 3 should be using x facts that they are familiar with, year 4 should be multiplying 2 and 3 digit numbers by a 1 digit number, year 5 should be multiplying up to 4 digit numbers by 2 and 1 digit numbers (including long multiplication for 2 digit numbers 23×25) and year 6 should be able to multiply numbers up to 4 digits by a 2 digit number using the formal method of long multiplication.

Grid method for short multiplication (multiplication by a single digit)

$$23 \times 8$$

x	20	3
8	160	24

$$\begin{array}{r} = 160 \\ + 24 \\ \hline 184 \end{array}$$

Multiplying a 2-digit number by a single digit number

$$153 \times 4$$

X	100	50	3
4	400	200	12

$$\begin{array}{r} = 400 \\ + 200 \\ + 12 \\ \hline 612 \end{array}$$

Extending to a 3-digit number multiplied by a single digit number

Formal written multiplication for short multiplication-expanded form

$$\begin{array}{r} 23 \\ \times 7 \\ \hline 21 \quad (3 \times 7) \\ 140 \quad (20 \times 7) \\ \hline 161 \end{array}$$

Children partition vertically to lead to a more formal recording.

Only when they are ready.

STAGE 5:

Grid method for short multiplication (multiplication by a single digit)

$$346 \times 9$$

x	300	40	6
9	2700	360	54

$$\begin{array}{r} 2700 \\ + 360 \\ + 54 \\ \hline 3114 \\ 11 \end{array}$$

Grid method for long multiplication (multiplication by a 2-digit)	<div>72×38</div> <table><tr><td>x</td><td>70</td><td>2</td></tr><tr><td>30</td><td>2100</td><td>60</td></tr><tr><td>8</td><td>560</td><td>16</td></tr></table> <div>$\begin{array}{r} 2100 \\ + 560 \\ + 60 \\ + 16 \\ \hline 2736 \\ 1 \end{array}$</div>	x	70	2	30	2100	60	8	560	16
x	70	2								
30	2100	60								
8	560	16								
Multiplying decimals using the grid method	<div>4.9×3</div> <table><tr><td>x</td><td>4</td><td>0.9</td></tr><tr><td>3</td><td>12</td><td>2.7</td></tr></table> <div>$\begin{array}{r} 12 \\ + 2.7 \\ \hline 14.7 \end{array}$</div>	x	4	0.9	3	12	2.7			
x	4	0.9								
3	12	2.7								
Formal written multiplication – expanded form 3 digit number by a single digit number	<div>$\begin{array}{r} 423 \\ \times 7 \\ \hline 21 \quad (3 \times 7) \\ 140 \quad (20 \times 7) \\ 2800 \quad (400 \times 7) \\ \hline 2961 \end{array}$</div>									

Grid method for short multiplication ThHTU x U	<div>4346 x 8</div> <table><tr><td>x</td><td>4000</td><td>300</td><td>40</td><td>6</td></tr><tr><td>8</td><td>32000</td><td>2400</td><td>320</td><td>48</td></tr></table>	x	4000	300	40	6	8	32000	2400	320	48	<div>32000 + 2400 + 320 + 48 <hr/>34768</div>		
x	4000	300	40	6										
8	32000	2400	320	48										
Grid method for long multiplication HTU x TU	<div>372 x 24</div> <table><tr><td>x</td><td>300</td><td>70</td><td>2</td></tr><tr><td>20</td><td>6000</td><td>1400</td><td>40</td></tr><tr><td>4</td><td>1200</td><td>280</td><td>8</td></tr></table>	x	300	70	2	20	6000	1400	40	4	1200	280	8	<div>6000 + 1400 + 1200 + 280 + 40 + 8 <hr/>8928 1</div>
x	300	70	2											
20	6000	1400	40											
4	1200	280	8											

Multiplying decimals using the grid method

$$4.92 \times 3$$

x	4	0.9	0.02
3	12	2.7	0.06

$$\begin{array}{r} 12 \\ + 0.7 \\ + 0.06 \\ \hline 12.76 \end{array}$$

Moving onto numbers with 2 decimal places when ready

Expanded form of short multiplication, moving onto the compact method once secure

Short Multiplication:

$$\begin{array}{r} 4346 \\ \times 8 \\ \hline 48 \\ 320 \\ 2400 \\ 32000 \\ \hline 34768 \end{array} \quad \longrightarrow \quad \begin{array}{r} 4346 \\ \times 8 \\ \hline 34768 \end{array}$$

leading to 2 3 4

Expanded form of long multiplication, moving onto the compact method once secure

Long Multiplication:

$$\begin{array}{r} 352 \\ \times 27 \\ \hline 14 \\ 350 \\ 2100 \\ 40 \\ 1000 \\ 6000 \\ \hline 9504 \\ 1 \end{array} \quad \longrightarrow \quad \begin{array}{r} 352 \\ \times 27 \\ \hline 2464 \\ 7040 \\ 9504 \\ 1 \end{array}$$

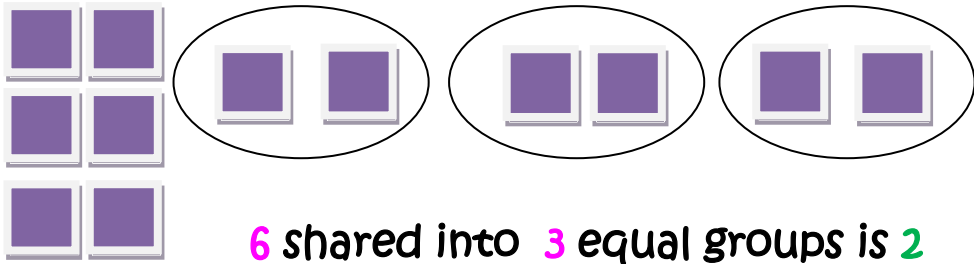
DIVISION

Key Vocabulary:

lots of groups of group share
shared between divide divide into
division divided by remainder
factor quotient divisible

EARLY STAGE 1:

At the early stage, the focus is primarily on practical experiences involving sharing items into equal groups with the use of concrete apparatus for the children to physically count and see.

Practical activities	 <p>6 shared into 3 equal groups is 2</p> <p><i>Understand division as sharing and grouping</i></p>
Pictorial representations	

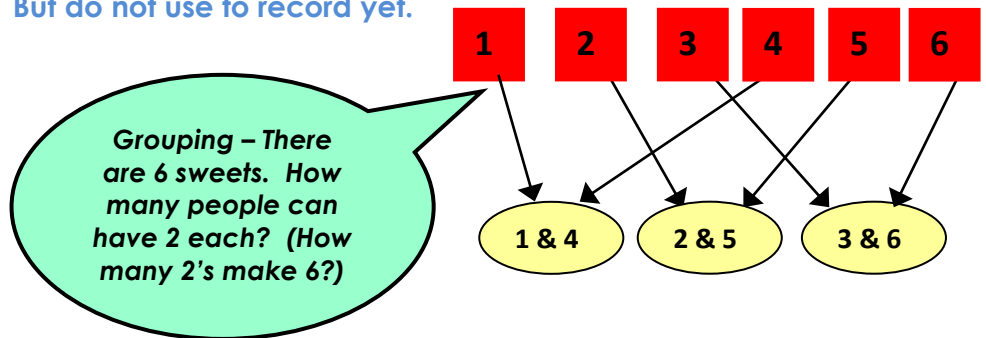
Key Stage 1

Emphasis in this stage is in counting in steps and the recall and use of multiplication facts. Children should be introduced to the division \div symbol and solve division problems using concrete objects, arrays, repeated subtraction and division facts.

Sharing equally
Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s and 3s.

Count forwards and backwards in 2s/ 5s/ 10s (3s in year 2 from any number).

Introduced to the \div sign $\rightarrow 6 \div 2$ can be modelled as:
But do not use to record yet.



Count forward \rightarrow Counting in 1s \leftarrow Count backward

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Counting in 2s

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Counting in 5s

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Count forward \downarrow
Count backward \uparrow

Counting in 10s

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Count forward \downarrow
Count backward \uparrow

Understand division as sharing and grouping

Sharing equally – 2's, 10's and 5's

Grouping using pictures

Begin using the division sign

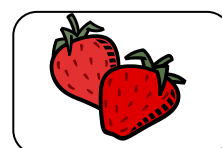
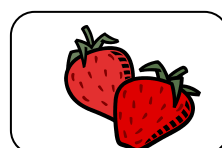
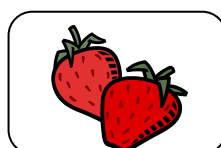
There are 6 strawberries, how many people can have 2 strawberries each?

OR

There are 6 strawberries, how many strawberries does each person get if there are 3 people.

3 people

2 strawberries each



Repeated subtraction using a prepared number

$$12 \div 3 = 4$$

4th
Group

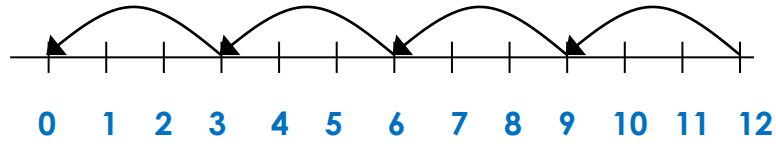
3rd
group

2nd
group

1st
group

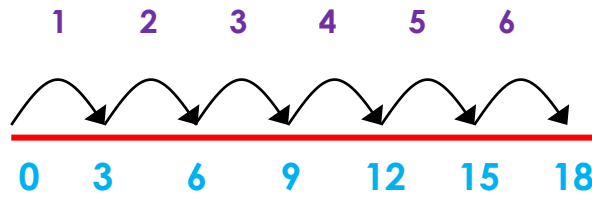
line

as $10 \div 5$ as 'how many 5s make 10?'



Start at 12, jump back 3 each time, add the amount of jumps to get the answer

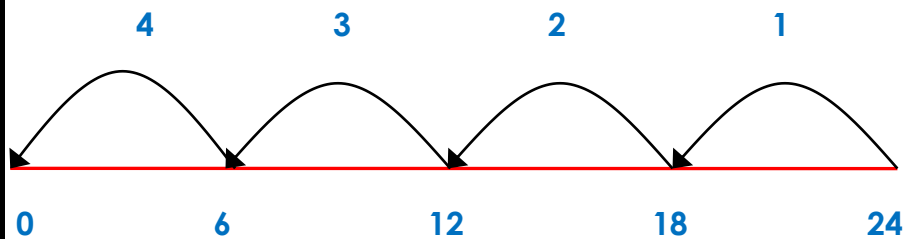
Emphasis on grouping rather than sharing using a blank number line



GROUPING: $18 \div 3$ as 'how many 3's make 18?'

Repeated subtraction using a number line

$$24 \div 6 = 4$$

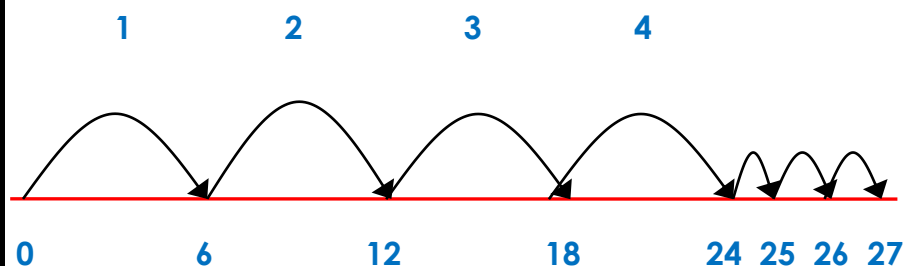


Children use blank number lines to support their understanding

Introducing remainders

$$27 \div 6 = 4 \text{ r } 3$$

Grouping – How many 6's make 26, how many left over?



Key Stage 2

At this stage children should start to use formal written methods for division, moving from the chunking method, to short division and long division. By year 5 children should be dividing numbers up to 4 digits by a 1 digit using the short division method. In year 6 they should be dividing numbers up to 4 digit by 2 digit whole numbers using short or long division. At this stage remainders need to be interpreted as either whole number remainders, fractions or rounding – according to context.

<p>Chunking method: 2 digit number divided by a single digit number</p> <p>Children begin with familiar multiples 2, 5, 10 and 3</p>	<p>$72 \div 3$</p> <div><div>$\begin{array}{r} 3 \overline{) 72} \\ - 30 \\ \hline 42 \\ - 30 \\ \hline 12 \\ - 6 \\ \hline 6 \\ - 6 \\ \hline 0 \end{array}$</div><div>$10 \times 3 = 30$ $10 \times 3 = 30$ $2 \times 3 = 6$ $2 \times 3 = 6$</div></div> <p>Follows the repeated subtraction method. Children take-away small chunks at a time until they get to zero.</p> <p>Answer : $10 + 10 + 2 + 2 = 24$</p>
<p>Leading to subtraction of other multiples</p>	<p>$96 \div 6$</p> <div><div>$\begin{array}{r} 16 \\ 6 \overline{) 96} \\ - 60 \\ \hline 36 \\ - 36 \\ \hline 0 \end{array}$</div><div>$10 \times 6$ 6×6</div></div> <p>Answer : $10 + 6 = 16$</p>

<p>Continue vertical written method (chunking) with and without remainders</p> <p>Subtracting larger multiples</p>	<p>$196 \div 6$</p> <div><div>$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{) 196} \\ - 180 \\ \hline 16 \\ - 12 \\ \hline 4 \end{array}$</div><div>$30 \times 6$ 2×6</div></div> <p>Children become more efficient and will now look for opportunities to reduce workings by subtracting larger multiples. e.g. Instead of subtracting 10×6 three times, children will subtract 30×6 and use known facts to assist, ie: 3×6</p> <p>Answer : $30 + 2 = 32$ with 4 left over $32 \text{ r } 4$</p>
--	---

Continue subtracting larger multiples

Continue chunking method to solve 3 digit numbers divided by a single digit number, extending to 3 digit numbers divided by 2-digit numbers

$$972 \div 36$$

$$\begin{array}{r} 27 \\ 36 \overline{) 972} \\ - 720 \\ \hline 252 \\ - 252 \\ \hline 0 \end{array}$$

20×36
 7×36

Answer: $20 + 7 = 27$

Using known facts: 2×26 or double $36 = 72$, adding the zero to make 720

Short division

Step 1: 6 into 19 = 3 times with 1 remaining. The 1 is carried over to make 16.

Step 2: 6 into 16 = 2 times with 4 remaining

$$\begin{array}{r} 32 \text{ r}4 \\ 6 \overline{) 196} \end{array}$$

step 1
step 2

Remainders as fractions

Step 1: 6 into 19 = 3 times with 1 remaining. The 1 is carried over to make 16.

Step 2: 6 into 16 = 2 times with 4 remaining. As a fraction $\frac{4}{6}$

Long division –
with remainders

432 ÷ 15 becomes

$$\begin{array}{r}
 28 \text{ r } 12 \\
 15 \overline{) 432} \\
 \underline{30 } \\
 132 \\
 \underline{120} \\
 12
 \end{array}$$

Answer: 28 remainder 12

Long division –
with fraction as
remainder.

$$\begin{array}{r}
 28 \\
 15 \overline{) 432} \\
 \underline{30 } \quad 15 \times 20 \\
 132 \\
 \underline{120} \quad 15 \times 8 \\
 12
 \end{array}$$

$$\frac{12}{15} = \frac{4}{5}$$

Answer: $28 \frac{4}{5}$

Long division
with decimal
remainders

$$\begin{array}{r}
 28.8 \\
 15 \overline{) 432.0} \\
 \underline{30 } \quad \downarrow \\
 132 \quad \downarrow \\
 \underline{120} \quad \downarrow \\
 120 \\
 \underline{120} \\
 0
 \end{array}$$

Answer: 28.8